# MATHEMATICS

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## XI<sup>th</sup>, XII<sup>th</sup>, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

## SCALAR OR DOT PRODUCT

& Their Properties

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#### THINGS TO REMEMBER

- If  $\vec{a}$  and  $\vec{b}$  are two non-zero vectors inclined at an angle  $\theta$ , then 1.
  - (i)  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$
  - (ii) Projection of  $\vec{a}$  on  $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \vec{a} \cdot \hat{b}$
  - (iii) Projection of  $\vec{b}$  on  $\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \vec{b} \cdot \hat{a}$
  - (iv) Projection of  $\vec{a}$  on  $\vec{b} = \left\{ \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right\} \hat{b} = \left\{ \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right\} \vec{b}$
  - (v) Projection vector  $\vec{b}$  on  $\vec{a} = \left\{ \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right\} \hat{a} = \left\{ \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right\} \vec{a}$
  - (vi)  $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$  is perpendicular to  $\vec{b}$ .
  - (vii)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
  - (viii)  $\vec{a} \cdot \vec{a} = |\vec{a}|^2$
  - (ix)  $m\vec{a} \cdot \vec{b} = m(\vec{a} \cdot \vec{b}) = \vec{a}$ ,  $m\vec{b}$ , for scalar m
  - (x)  $m\vec{a} \cdot n\vec{b} = mn(\vec{a} \cdot \vec{b}) = mn(\vec{a} \cdot \vec{b}) = \vec{a} \cdot mn\vec{b}$  for scalars m, n
  - $(xi) |\vec{a} \pm \vec{b}| \le |\vec{a}| + |\vec{b}|$
  - $(xii)|\vec{a} \square \vec{b}| \ge |\vec{a}| |\vec{b}|$
  - (xiii)  $|\vec{a} \Box + \vec{b}|^2 \ge |\vec{a}|^2 |\vec{b}|^2 \pm 2(\vec{a} \cdot \vec{b})$
  - (xiv)  $(\vec{a} + \vec{b}) \cdot (\vec{a} \vec{b}) = |\vec{a}|^2 |\vec{b}|^2$
  - (xv)  $\vec{a} \cdot \vec{b} > 0$  if  $\theta$  is acute
  - (xvi)  $\vec{a} \cdot \vec{b} < 0$  if  $\theta$  is obtuse
- If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three vectors, then  $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ 2.  $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$
- If  $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$  and  $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ , then  $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ . 3.
- If  $\vec{a}$  and  $\vec{b}$  are two vectors inclined at an angle  $\theta$ , then  $\cos \theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ 4.

5. If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are non-coplanar vectors in space and  $\vec{r}$  is any vector in space, then  $\vec{r} = (\vec{r} \cdot \hat{a}) \hat{a} + (\vec{r} \cdot \hat{b}) \hat{b} + (\vec{r} \cdot \hat{c}) \hat{c}$ 

where  $\hat{a}, \hat{b}, \hat{c}$  are unit vectors in the directions of  $\vec{a}, \vec{b}, \vec{c}$  respectively.

Also, 
$$\vec{r} = \left\{ \frac{\vec{r} \cdot \vec{a}}{|\vec{a}|^2} \right\} \vec{a} + \left\{ \frac{\vec{r} \cdot \vec{b}}{|\vec{b}|^2} \right\} \vec{b} + \left\{ \frac{\vec{r} \cdot \vec{c}}{|\vec{b}|^2} \right\} \vec{c}$$

In particular,  $\vec{r} = (\vec{r} \cdot \hat{i}) \hat{i} + (\vec{r} \cdot \hat{j}) \hat{j} + (\vec{r} \cdot \hat{k}) \hat{k}$ .

#### **EXERCISE-1**

- 1. The scalar product of two vectors is commutative i.e.  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ .
- 2. For any two vectors  $\vec{a}$  and  $\vec{b}$ , we have

(i) 
$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

(ii) 
$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

(iii) 
$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$$

- 3. Find  $\vec{a} \cdot \vec{b}$  when,
  - (i)  $\vec{a} = 2\hat{i} + 2\hat{j} \hat{k}$  and  $\vec{b} = 6\hat{i} 3\hat{j} + 2\hat{k}$
  - (ii)  $\vec{a} = (1, 1, 2)$  and  $\vec{b} = (3, 2, -1)$
- 4. Find  $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} \vec{b})$ , if  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 2\hat{j} \hat{k}$ .
- 5. For any vector  $\vec{r}$ , prove that  $\vec{r} = (\vec{r} \cdot \hat{i})\hat{i} + (\hat{i} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k}$ .
- 6. Dot products of a vector with vectors  $3\hat{i} 5\hat{k}$ ,  $2\hat{i} + 7\hat{j} + \hat{k}$  are respectively -1, 6 and 5. Find the vector.
- 7. Find the value of  $\lambda$  so that the vector  $\vec{a} = 2\hat{i} + \lambda \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} 2\hat{j} + 3\hat{k}$  are perpendicular to each other.
- 8. Find the value of p for which the vector  $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$  and  $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$  are
  - (i) perpendicular
  - (ii) Parallel.
- 9. Find the angle between two vector  $\vec{a}$  and  $\vec{b}$  having the same length  $\sqrt{2}$  and their sealar product is -1.
- 10. Find the angle between the vectors  $5\hat{i} + 3\hat{j} + 4\hat{k}$  and  $6\hat{i} 8\hat{j} \hat{k}$ .

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- If  $\hat{i} + \hat{j} + \hat{k}$ ,  $2\hat{i} + 5\hat{j}$ ,  $3\hat{i} + 2\hat{j} 3\hat{k}$  and  $\hat{i} 6\hat{j} \hat{k}$  respectively are the position vectors of pointing A, B, C and D, then find the angle between the straight lines AB and CD. Deduce that AB and CD are collinear.
- Find the projection of the vector  $7\hat{i} + \hat{j} 4\hat{k}$  on  $2\hat{i} + 6\hat{j} + 3\hat{k}$ . 12.
- Show that the projection vector of  $\vec{a}$  on  $\vec{b}$  ( $\neq \vec{0}$ ) (component of  $\vec{a}$  along  $\vec{b}$ ) is  $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\vec{b}$ . 13.
- Show that the projection vector of  $\vec{b}$  on  $\vec{a} \neq 0$  is  $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)^{\vec{a}}$ .
- 15. For any two vectors  $\vec{a}$  and  $\vec{b}$ , prove that:

(i) 
$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

(ii) 
$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

(iii) 
$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$$

Interpret the result geometrically.:

(iv) 
$$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow \vec{a} \perp \vec{b}$$

Interpret the result geometrically. :

(v) 
$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \Rightarrow \vec{a}$$
 is parallel to  $\vec{b}$ 

(vi) 
$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \Rightarrow \vec{a}, \vec{b}$$
 are orthogonal.

- For any two vectors  $\vec{a}$  and  $\vec{b}$ , prove that  $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$
- Prove Cauchy-Schawarz inequality  $(\vec{a} \cdot \vec{b})^2 \le |\vec{a}|^2 |\vec{b}|^2$ 17.
- If two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = 2$ ,  $|\vec{b}| = 1$  and  $\vec{a} \cdot \vec{b} = 1$ , find  $(3\vec{a} 5\vec{b}) \cdot (3\vec{a} + 7\vec{b})$ .
- Find  $|\vec{x}|$ , if for a unit vector  $\vec{a}$ ,  $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 15$ . 19.
- The scalar product of the vector  $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$  with a unit vector along the sum of the vectors  $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} -$ 20.  $5\,\hat{k}$  and  $\lambda\,\hat{i}+2\,\hat{j}+3\,\hat{k}$  is equal to 1. Find the value of  $\lambda$ .
- If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ ,  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$  and  $|\vec{c}| = 7$ , find the angle between  $\vec{a}$  and  $\vec{b}$ .
- If  $\vec{a} = 5\hat{i} \hat{j} 3\hat{k}$ , and  $\vec{b} = \hat{i} + 3\hat{j} 5\hat{k}$ , then show that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} \vec{b}$  are prependicular.
- Two vector  $\vec{a}$  and  $\vec{b}$ , prove that the vector  $|\vec{a}| |\vec{b} + |\vec{b}| |\vec{a}|$  is orthogonal to the vector  $|\vec{a}| |\vec{b} |\vec{b}| |\vec{a}|$ .
- Show that the vectors  $2\hat{\mathbf{i}} \hat{\mathbf{j}} + \hat{\mathbf{k}}$ ,  $\hat{\mathbf{i}} 3\hat{\mathbf{j}} 5\hat{\mathbf{k}}$ ,  $3\hat{\mathbf{i}} 4\hat{\mathbf{j}} 4\hat{\mathbf{k}}$  form the sides of a right angled triangle.

- Find the value of x for which the angle between the vectors  $\vec{a} = 2x^2 \hat{i} + 4x \hat{j} + \hat{k}$  and  $\vec{b} = 7\hat{i} 2\hat{j} + x\hat{k}$ 25. is obtuse.
- Find the value of a for which the vectors  $\vec{a} = (c \log_2 x)\hat{i} 6\hat{j} + 3\hat{k}$  and  $\vec{b} = (\log_2 x)\hat{i} + 2\hat{j} + (2c \log_2 x)\hat{i}$ x)  $\hat{k}$  make an abtuse angle for any  $x \in (0, \infty)$ .
- If  $\vec{a}$ ,  $\vec{b}$  are two vectors such that  $|\vec{a} + \vec{b}| = |\vec{a}|$ , then prove that  $2\vec{a} + \vec{b}$  is prependicular to  $\vec{b}$ .
- If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three mutually prependicular vectors of equal magnitude, prove that  $\vec{a} + \vec{b} + \vec{c}$  is equally inclined with vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .
- Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  be three vectors of magnitudes 3, 4 and 5 respectively. If each one is perpendicular to the sum of the other two vectors, prove that  $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$ .
- If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , find the value of  $\vec{a} \cdot \vec{b} \cdot \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ .
- Three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  satisfy the condition  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ . Evaluate the quantity  $\mu \vec{a} \cdot \vec{b} \cdot \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ , if  $|\vec{a}| = 1$ ,  $|\vec{b}| = 4$  and  $|\vec{c}| = 2$ .
- Dot product of a vector with  $\hat{\mathbf{i}} + \hat{\mathbf{j}} 3\,\hat{\mathbf{k}}$ ,  $\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} 2\,\hat{\mathbf{k}}$  and  $2\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$  are C, 5 and 8 respectively. Find the vector.
- If  $\vec{a}$  and  $\vec{b}$  are unit vectors inclined at an angle  $\theta$ , then prove that

(i) 
$$\cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}|$$

(ii) 
$$\tan \frac{\theta}{2} = \frac{|\hat{\mathbf{a}} - \hat{\mathbf{b}}|}{|\hat{\mathbf{a}} + \hat{\mathbf{b}}|}$$

- If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three mutually perpendicular unit vectors, then prove that  $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$ .
- Show that the vectors  $\vec{a} = \frac{1}{7} (2\hat{i} + 3\hat{j} + 6\hat{k}), \ \vec{b} = \frac{1}{7} (3\hat{i} 6\hat{j} + 2\hat{k}), \ \vec{c} = \frac{1}{7} (6\hat{i} + 2\hat{j} + 3\hat{k})$  are mutually 35. perpendicular unit vectors.
- If  $|\vec{a}| = a$  and  $|\vec{b}| = b$ , prove that  $\left(\frac{\vec{a}}{a_2} \frac{\vec{b}}{b_2}\right) = \left(\frac{\vec{a} \vec{b}}{ab}\right)$ .
- Show that the vectors  $\vec{a} = 3\hat{i} 2\hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} 3\hat{j} + 5\hat{k}$ ,  $\vec{c} = 2\hat{i} + \hat{j} 4\hat{k}$  form a right angled triangle.
- If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , show that the angle  $\theta$  between the vectors  $\vec{b}$  and  $\vec{c}$  is given by 38.

$$\cos \theta = \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|}.$$

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- 39. Let  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$  be vector such  $\vec{u} + \vec{v} + \vec{w} = \vec{0}$ . If  $|\vec{u}| = 3$ ,  $|\vec{v}| = 4$  and  $|\vec{w}| = 5$ , then find  $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}}$
- Show that the points whose position vectors are  $\vec{a} = 4\hat{i} 3\hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} 4\hat{j} + 5\hat{k}$ ,  $\vec{c} = \hat{i} \hat{j}$  form a right triangle.
- 41. Evaluate:  $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + \vec{b})$
- If  $\vec{a}$  is a unit vector, then find  $|\vec{x}|$  in each of the following:
  - (i)  $(\vec{x} \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$
  - (ii)  $(\vec{x} \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$
- 43. Find  $|\vec{a}|$  and  $|\vec{b}|$ , if
  - (i)  $(\vec{a} + \vec{b}) \cdot (\vec{a} \vec{b}) = 12$  and  $|\vec{a}| = 2|\vec{b}|$
  - (ii)  $(\vec{a} + \vec{b}) \cdot (\vec{a} \vec{b}) = 8$  and  $|\vec{a}| = 8 |\vec{b}|$
  - (iii)  $(\vec{a} + \vec{b}) \cdot (\vec{a} \vec{b}) = 3$  and  $|\vec{a}| = 2 |\vec{b}|$
- Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$ , if
  - (i)  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = \sqrt{6}$
  - (ii)  $|\vec{a}| = 3$ ,  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$
- Express the vector  $\vec{a} = 5\hat{i} 2\hat{j} + 5\hat{k}$  as the sum of two vectors such that one is parallel to the vector  $\vec{b} = 3\hat{i} + \hat{k}$  and other is prependicular to  $\vec{b}$ .
- If  $\vec{a}$  and  $\vec{b}$  are two vectors of the same magnitude inclined at an angle of 30° such that  $\vec{a} \cdot \vec{b} = 3$ , find  $|\vec{a}|$ ,  $|\vec{b}|$ .
- Decompose the vector  $6\hat{i}-3\hat{j}-6\hat{k}$  into vectors which are parallel and prependicular to the vector  $\hat{i} + \hat{i} + \hat{k}$ .
- If  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$ , what can you conclude about the vector  $\vec{b}$ ?
- A unit vector  $\vec{a}$  makes angles  $\frac{\pi}{2}$  and  $\frac{\pi}{3}$  with  $\hat{i}$  and  $\hat{j}$  respectively and an acute angle  $\theta$  with  $\hat{k}$ . Find the angle  $\theta$  and component of  $\vec{a}$ .
- If  $\vec{a} = 5\hat{i} \hat{j} 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} 5\hat{k}$ , then show that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} \vec{b}$  are orthogonal.
- Find the projection of  $\vec{b} + \vec{c}$  on  $\vec{a}$ , where  $\vec{a} = 2\hat{i} 2\hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} 2\hat{k}$  and  $\vec{c} = 2\hat{i} \hat{i} + 4\hat{k}$ .
- Find the magnitude of two vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude and such that the angle between them is 60° and their scalar product is 1/2.

- 53. If  $\vec{a} = 2\hat{i} 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda \vec{b}$  is perpendicular to c, then find the value of  $\lambda$ .
- If either  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \cdot \vec{b} = 0$ . But the converse need not be true. Justify your answer with an example.

#### **EXERCISE-2**

- What is the angle between vector  $\vec{a}$  and  $\vec{b}$  with magnitudes 2 and  $\sqrt{3}$  respectively? Given 1.  $\vec{a} \cdot \vec{b} = \sqrt{3}$ .
- If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $\vec{a} \cdot \vec{b} = 6$ ,  $|\vec{a}| = 3$  and  $|\vec{b}| = 4$ . Write the projection of  $\vec{a}$  on  $\vec{b}$ . 2.
- If the vectors  $3\hat{i} 2\hat{j} 4\hat{k}$  and  $18\hat{i} 12\hat{j} m\hat{k}$  are parallel, find the value of m. 3.
- For any two vector  $\vec{a}$  and  $\vec{b}$  write when  $|\vec{a} + \vec{b}| = |\vec{a} \vec{b}|$  holds. 4.
- If  $\vec{a}$ ,  $\vec{b}$  are unit vectors such that  $\hat{a} + \hat{b}$  is a unit vector, write the value of  $|\hat{a} \hat{b}|$ . 5.
- If  $|\vec{a}| = 2$ ,  $|\vec{b}| = 5$  and  $\vec{a} \cdot \vec{b} = 2$ , find  $|\vec{a} \vec{b}|$ . 6.
- For any two non-zero vector, write the value of  $\frac{|\vec{a} + \vec{b}|^2 + |\vec{a} \vec{b}|^2}{|\vec{a}|^2 + |\vec{b}|^2}$ . 7.
- Find the value of  $\theta \in (\theta, \pi/2)$  for which vectors  $\vec{a} = (\sin \theta) \hat{i} + (\cos \theta) \hat{j}$  and  $\vec{b} = \hat{i} \sqrt{3} \hat{j} + 2\hat{k}$  are 8. perpendicular.
- If  $\vec{a}$  and  $\vec{b}$  are matually perpendicular unit vector, write the value of  $|\vec{a} + \vec{b}|$ . 9.
- Find the angle between the vectors  $\vec{a} = \hat{i} \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} \hat{k}$ . 10.
- For what value of  $\lambda$  are the vectors  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} 2\hat{j} + 3\hat{k}$  perpendicular to each other? 11.
- Find the projection of  $\vec{a}$  on  $\vec{b}$  if  $\vec{a} \cdot \vec{b} = 8$  and  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ .
- Write the value of p for which  $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$  and  $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$  are parallel vectors. 13.
- Find the value of  $\lambda$  if the vectors  $2\hat{i} + \lambda \hat{j} + 3\hat{k}$  and  $3\hat{i} + 2\hat{j} 4\hat{k}$  are perpendicular to each other. 14.
- If  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 3$ , find the projection of  $\vec{b}$  on  $\vec{a}$ . 15.

### **EXERCISE-3**

- Let  $\vec{a}$  and  $\vec{b}$  be two vectors and  $\alpha$  be the angle between them, then  $\vec{a} + \vec{b}$  is a unit vector, if 1.
  - (a)  $\alpha = \frac{\pi}{4}$
- (b)  $\alpha = \frac{\pi}{2}$
- (c)  $\alpha = \frac{2\pi}{3}$
- (d)  $\alpha = \frac{\pi}{2}$

2. The vector component of  $\vec{b}$  prependicular to  $\vec{a}$  is

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- (a)  $(\vec{b} \cdot \vec{c})\vec{a}$
- (b)  $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{a}|^2}$
- (c)  $\vec{a} \times (\vec{b} \times \vec{a})$
- (d) none of these
- 3. If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are any three mutually perpendicular vectors of equal magnitude a, then  $|\vec{a} + \vec{b} + \vec{c}|$  is equal to
  - (a) a

- (b)  $\sqrt{2}$  a
- (c)  $\sqrt{3}$  a
- (d) none of these
- 4. The projection of the vector  $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$  along the vector of  $\hat{\mathbf{j}}$  is
  - (a) 1

(b) 0

(c) 2

(d) -1

- (e) -2
- 5. If  $\vec{a}$  and  $\vec{b}$  are unit vectors, then the greatest value of  $\sqrt{3} |\vec{a} + \vec{b}| + |\vec{a} \vec{b}|$  is
  - (a) 2

- (b)  $2\sqrt{2}$
- (c) 4

(d) none of these

- 6. The orthogonal projection of  $\vec{a}$  on  $\vec{b}$  is
  - (a)  $\frac{(\vec{a}.\vec{b})\vec{a}}{|\vec{a}|^2}$
- (b)  $\frac{\left(\vec{a}.\vec{b}\right)\vec{b}}{\left|\vec{b}\right|^2}$
- (c)  $\frac{\overline{a}}{|\overline{a}|^2}$
- (d)  $\frac{\vec{b}}{|\vec{a}|^2}$